

Autumn Block 4

Fractions A

Small steps

Step 1 Find fractions equivalent to a unit fraction

Step 2 Find fractions equivalent to a non-unit fraction

Step 3 Recognise equivalent fractions

Step 4 Convert improper fractions to mixed numbers

Step 5 Convert mixed numbers to improper fractions

Step 6 Compare fractions less than 1

Step 7 Order fractions less than 1

Step 8 Compare and order fractions greater than 1

Small steps

Step 9 Add and subtract fractions with the same denominator

Step 10 Add fractions within 1

Step 11 Add fractions with total greater than 1

Step 12 Add to a mixed number

Step 13 Add two mixed numbers

Step 14 Subtract fractions

Step 15 Subtract from a mixed number

Step 16 Subtract from a mixed number – breaking the whole

Small steps

Step 17

Subtract two mixed numbers

Find fractions equivalent to a unit fraction

Notes and guidance

Children are familiar with the idea of equivalent fractions from earlier study. This small step focuses on how unit fractions can be expressed in other forms.

It is important that children use a variety of representations, including fractions of shapes, number lines and fraction walls as well as the abstract form, so that they understand the relationships. They complement this conceptual understanding by using their times-table knowledge to find missing numerators or denominators, working both horizontally and vertically.

Children move on to find fractions equivalent to non-unit fractions in the next step and use this learning throughout the block.

Things to look out for

- Children may not understand that different fractions that represent the same amount are equivalent, for example $\frac{1}{3}$ and $\frac{2}{6}$
- Children may over-generalise to “do the same to the numerator and denominator” and use incorrect additive instead of multiplicative relationships, for example $\frac{1}{4} = \frac{2}{5}$ because $1 + 1 = 2$ and $4 + 1 = 5$

Key questions

- What does “equivalent” mean?
- What is a unit fraction?
- When are two fractions equivalent?
- How can you use the model to see if the two fractions are equivalent?
- How do you use a fraction wall to find equivalent fractions?
- What multiplication/division facts can you use?

Possible sentence stems

- A fraction is a unit fraction if the _____ is equal to _____
- The numerator has been multiplied/divided by _____, so if the denominator is multiplied/divided by _____, then the fractions will be equivalent.
- The denominator is _____ times the numerator in both fractions, so the fractions are _____

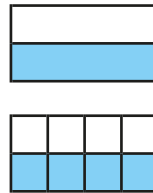
National Curriculum links

- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

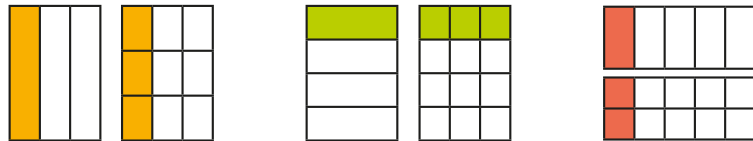
Find fractions equivalent to a unit fraction

Key learning

- Take two pieces of paper that are the same size. Fold one piece into 2 equal parts and the other piece into 8 equal parts. Explain how the pieces of paper show that $\frac{1}{2} = \frac{4}{8}$. Use more pieces of paper to find other fractions equivalent to one half.

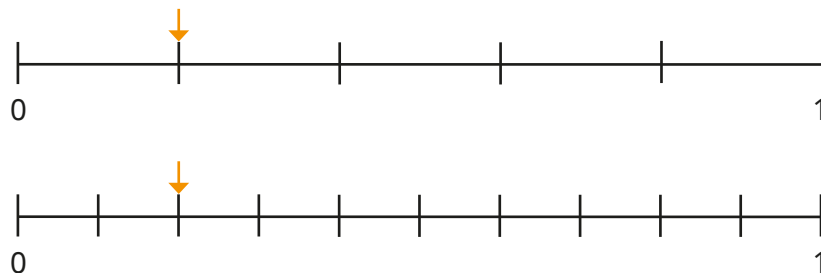


- Use the models to write equivalent fractions.

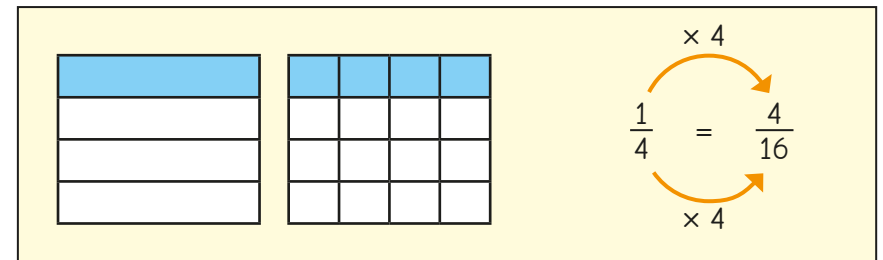


Make or draw more models to show other fractions equivalent to unit fractions.

- How do the number lines show that $\frac{1}{5}$ is equivalent to $\frac{2}{10}$?



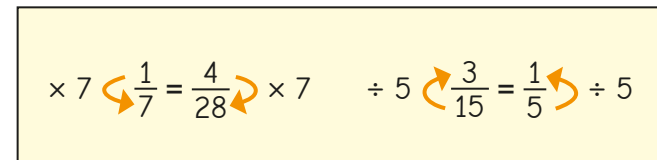
- Whitney uses diagrams and multiplication and division skills to find equivalent fractions.



Use Whitney's method to find the missing numbers.

$\frac{1}{2} = \frac{\square}{12}$
 $\frac{1}{5} = \frac{\square}{30}$
 $\frac{4}{\square} = \frac{1}{3}$
 $\frac{\square}{6} = \frac{5}{30}$

- Amir uses the relationships between the numerators and denominators to find equivalent fractions.



Use Amir's method to find the missing numbers.

$\frac{1}{2} = \frac{8}{\square}$
 $\frac{1}{5} = \frac{\square}{20}$
 $\frac{1}{12} = \frac{\square}{36}$
 $\frac{\square}{3} = \frac{10}{30}$

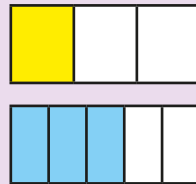
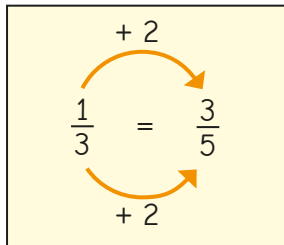
Find fractions equivalent to a unit fraction

Reasoning and problem solving

Tiny is finding fractions that are equivalent to one third.



I have done the same to the numerator and denominator.



$$\frac{1}{3} = \frac{2}{6}$$



Use a diagram to show that Tiny is wrong.

How many sixths are equivalent to one third?

Are the statements true or false?

$$\frac{1}{2} = \frac{10}{20}$$

$$\frac{1}{3} = \frac{15}{30}$$

$$\frac{1}{4} = \frac{40}{400}$$

$$\frac{1}{5} = \frac{20}{100}$$

$$\frac{1}{6} = \frac{12}{66}$$

$$\frac{1}{7} = \frac{4}{10}$$

True False
False True
False False

Explain your answers.



Complete the set of equivalent fractions.



$$\frac{1}{6} = \frac{\square}{12} = \frac{\square}{18} = \frac{4}{\square} = \frac{\square}{30} = \frac{6}{\square} = \frac{7}{\square}$$

2, 3, 24, 5, 36, 42

Find fractions equivalent to a non-unit fraction

Notes and guidance

Building from the previous step, in this small step children find fractions that are equivalent to a non-unit fraction.

Children continue to use a variety of representations, including fractions of shapes, number lines and parts of a fraction wall as well as the abstract form, to understand the relationships. They complement this conceptual understanding by using multiplication and division facts to find missing numerators or denominators when working in the abstract.

The understanding gained in this and the previous step will help children to recognise equivalent fractions in the next step and prepare them for when they add and subtract fractions with different denominators later in the block.

Things to look out for

- Children may not understand that different fractions that represent the same amount are equivalent, for example $\frac{2}{3}$ and $\frac{4}{6}$
- Children may over-generalise to “do the same to the numerator and denominator” and use incorrect additive instead of multiplicative relationships, for example $\frac{3}{4} = \frac{4}{5}$ because $3 + 1 = 4$ and $4 + 1 = 5$

Key questions

- What does “equivalent” mean?
- When are two fractions equivalent?
- How can you use the diagram to see if the two fractions are equivalent?
- How can you use your knowledge about unit fractions to help with non-unit fractions?
- How do you use a fraction wall to find equivalent fractions?
- What multiplication/division facts can you use?

Possible sentence stems

- The numerator has been multiplied/divided by _____, so if the denominator is multiplied/divided by _____, then the fractions will be equivalent.
- I know that _____ is equivalent to _____ because ...

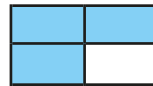
National Curriculum links

- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

Find fractions equivalent to a non-unit fraction

Key learning

- Take two pieces of paper that are the same size.



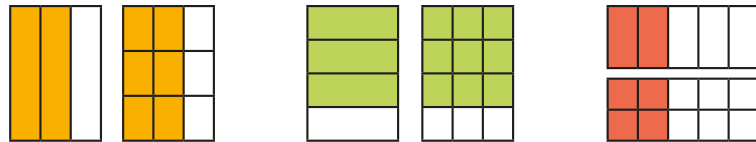
Fold one piece into 4 equal parts and the other piece into 8 equal parts.



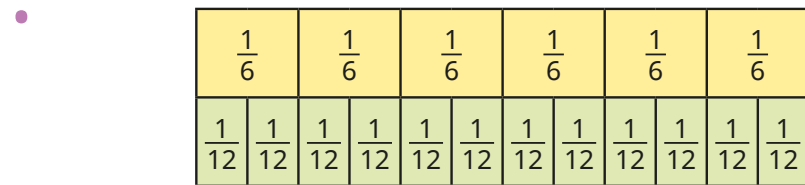
Explain how the pieces of paper show $\frac{3}{4} = \frac{6}{8}$

Use more pieces of paper to find other fractions equivalent to $\frac{3}{4}$

- Use the diagrams to write equivalent fractions.



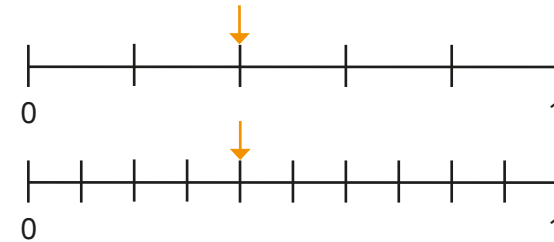
Make or draw more diagrams to show other fractions equivalent to non-unit fractions.



Use the bar model to complete the equivalent fractions.

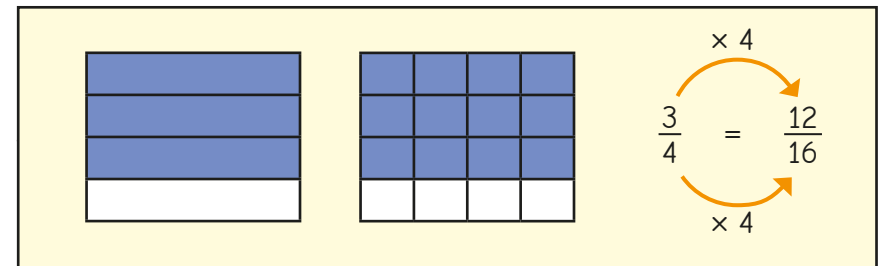
▶ $\frac{2}{6} = \frac{\square}{12}$ ▶ $\frac{3}{6} = \frac{\square}{12}$ ▶ $\frac{4}{6} = \frac{\square}{12}$ ▶ $\frac{5}{6} = \frac{\square}{12}$ ▶ $\frac{6}{6} = \frac{\square}{12}$

- How do the number lines show that $\frac{2}{5}$ is equivalent to $\frac{4}{10}$?



What other equivalent fractions can be seen from the number lines?

- Scott uses diagrams and multiplication and division skills to find equivalent fractions.



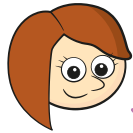
Use Scott's method to find the missing numbers.

▶ $\frac{3}{5} = \frac{\square}{20}$ ▶ $\frac{4}{9} = \frac{\square}{27}$ ▶ $\frac{16}{\square} = \frac{2}{3}$ ▶ $\frac{\square}{6} = \frac{25}{30}$

Find fractions equivalent to a non-unit fraction

Reasoning and problem solving

Rosie is working out some equivalent fractions.



To find equivalent fractions, whatever you do to the numerator, you do to the denominator.

A

$$\frac{4}{8} = \frac{8}{16}$$

B

$$\frac{4}{8} = \frac{6}{10}$$

C

$$\frac{4}{8} = \frac{2}{4}$$

D

$$\frac{4}{8} = \frac{1}{5}$$

Which of Rosie's equivalent fractions are correct?

For any incorrect answers, explain the mistake Rosie has made.



A and C

Here are some fraction cards.



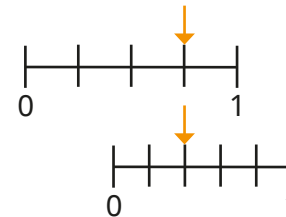
$\frac{4}{A}$	$\frac{B}{C}$	$\frac{20}{50}$
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Use the clues to work out the values of A, B and C.

- All three fractions are equivalent.
- $A + B = 16$

A = 10, B = 6,
C = 15

Tiny thinks that the number lines show that $\frac{3}{4}$ is equivalent to $\frac{2}{5}$



No

Is Tiny correct?

Explain your answer.



Recognise equivalent fractions

Notes and guidance

Children develop their learning from the previous two steps to recognise pairs and larger sets of equivalent fractions.

Various methods are explored, including looking for common factors and multiples to establish whether fractions are equivalent, and also looking at the multiplicative relationship between the numerator and denominator. The use of diagrams and other pictorial representations are used throughout to support children's understanding of the abstract methods.

The key point of this step is to recognise equivalent fractions, and although this includes some simplifying, there is no need to focus on writing fractions in their simplest form, which is covered in Year 6

Things to look out for

- Errors may occur in finding the common factors of the numerator and denominator.
- Children may get confused when looking for the relationships between numerators and denominators.
- Children may over-generalise to “do the same to the numerator and denominator” and use incorrect additive instead of multiplicative relationships, for example $\frac{3}{4} = \frac{4}{5}$ because $3 + 1 = 4$ and $4 + 1 = 5$

Key questions

- What does “equivalent” mean?
- When are two fractions equivalent?
- How can you use a fraction wall to check if the fractions are equivalent?
- What are the common factors of the numerator and the denominator?
- Are there any other factors you could use?
- What is the relationship between the numerator and the denominator of the fractions?

Possible sentence stems

- _____ is a common factor of the numerator and the denominator, so I can divide both of these by _____ to find an equivalent fraction.
- The numerator/denominator has been multiplied by _____, so the denominator/numerator should also be _____ by _____

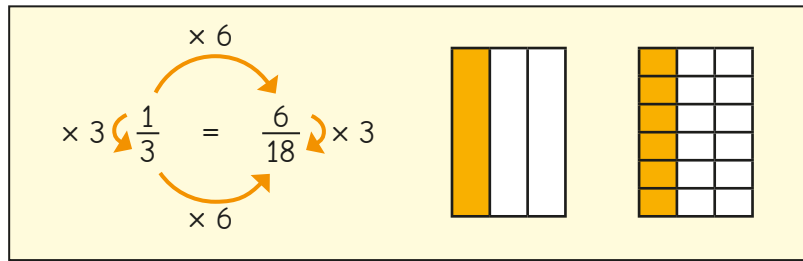
National Curriculum links

- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

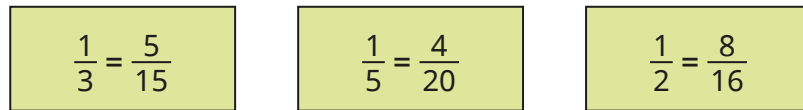
Recognise equivalent fractions

Key learning

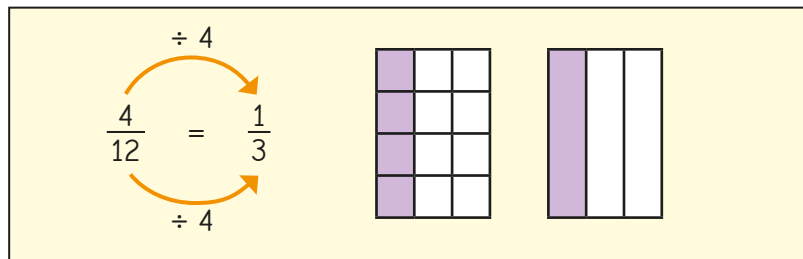
- Alex has shown that $\frac{1}{3}$ is equivalent to $\frac{6}{18}$



Show that the fractions are equivalent.



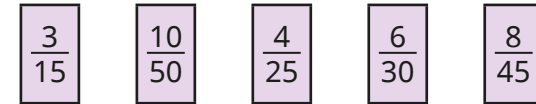
- Mo has shown that $\frac{4}{12}$ is equivalent to $\frac{1}{3}$



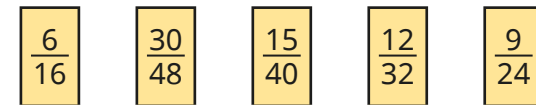
Show that the fractions are equivalent.



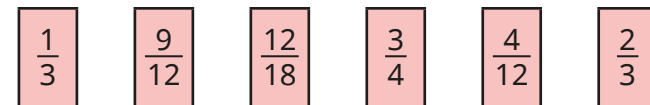
- Use multiplication or division facts alongside diagrams to decide which of the fractions are equivalent to $\frac{1}{5}$



- Use multiplication or division facts alongside diagrams to decide which of the fractions are equivalent to $\frac{3}{8}$

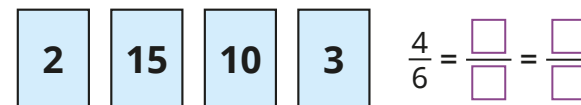


- Here are six fractions.



Sort the fractions into three pairs of equivalent fractions.

- Use the number cards to complete the equivalent fractions.

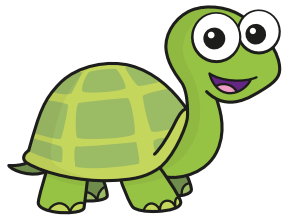


Recognise equivalent fractions

Reasoning and problem solving

Tiny is working out equivalent fractions.

$\frac{12}{20}$ and $\frac{30}{50}$ are equivalent fractions.



Show that Tiny is correct.



Both fractions are equivalent to $\frac{3}{5}$

Are the fractions equivalent?

$$\frac{30}{40}$$

$$\frac{36}{48}$$

Explain your answer.



Yes

Sam and Eva find out that $\frac{18}{48}$ is equivalent to $\frac{3}{8}$



Here are Sam's workings.

$$\frac{18}{48} \xrightarrow{\div 2} \frac{9}{24} \xrightarrow{\div 3} \frac{3}{8}$$

Here are Eva's workings.

$$\frac{18}{48} \xrightarrow{\div 6} \frac{3}{8}$$

Eva's

multiple possible answers

Whose method is more efficient?

Explain your answer.



Show that $\frac{60}{144}$ is equivalent to $\frac{5}{12}$

Convert improper fractions to mixed numbers

Notes and guidance

Children encountered fractions greater than 1 and mixed numbers in Year 4

They may need reminding that an improper fraction is one where the numerator is greater than or equal to the denominator and a mixed number consists of an integer and a proper fraction.

Children use objects and diagrams to make wholes to support converting improper fractions into mixed numbers. Once they are confident with this as a concept, they move on to a more abstract approach using division and remainders. Understanding the whole is key to their understanding.

This skill is important for adding fractions and adding mixed numbers later in the block.

Things to look out for

- Children may not make connections between the denominator of a fraction and the number of equal parts needed to make one whole.
- Children may have the misconception that a fraction is always “part of a whole” and not realise that an improper fraction is a fraction that is greater than or equal to 1

Key questions

- How many _____ are there in one whole?
- How many _____ are there in $2/3/4$ wholes?
- What does each part of a mixed number represent?
- What is an improper fraction?
- How many cubes do you need to represent the improper fraction? How can you use the cubes to make wholes? What do the remaining cubes represent?

Possible sentence stems

- There are _____ in one whole, so there are _____ in _____ wholes.
- I can regroup _____ to make _____ wholes with _____ parts left over. As a mixed number, this is _____ and _____

National Curriculum links

- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Convert improper fractions to mixed numbers

Key learning

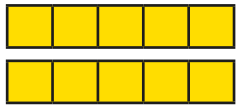
- The bar model shows that $1 = \frac{6}{6}$



Work out the missing numbers.

▶ $1 = \frac{5}{\square}$ ▶ $1 = \frac{7}{\square}$ ▶ $1 = \frac{\square}{8}$ ▶ $\frac{\square}{4} = 1$

- The bar models shows that $2 = \frac{10}{5}$

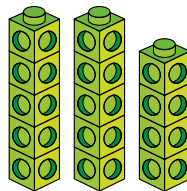


Work out the missing numbers.

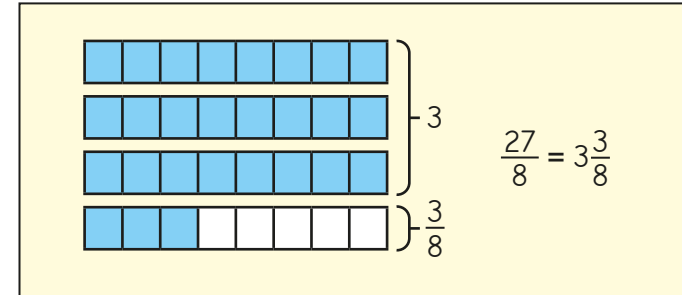
▶ $3 = \frac{\square}{5}$ ▶ $2 = \frac{\square}{4}$ ▶ $\text{---} = \frac{16}{4}$ ▶ $\frac{9}{3} = \text{---}$

- Nijah converts $\frac{14}{5}$ to a mixed number using cubes.

- ▶ Why has Nijah built towers of 5 cubes?
- ▶ How many full towers of 5 has Nijah built?
- ▶ How many cubes are in the last tower?
- ▶ Write $\frac{14}{5}$ as a mixed number.

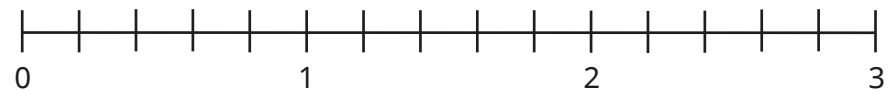


- Tommy uses a bar model to convert the improper fraction $\frac{27}{8}$ to a mixed number.



Use Tommy's method to convert $\frac{25}{8}$, $\frac{26}{8}$, $\frac{18}{7}$ and $\frac{19}{4}$ to mixed numbers.

- Label $\frac{7}{5}$ and $\frac{13}{5}$ on the number line. Write each fraction as a mixed number.



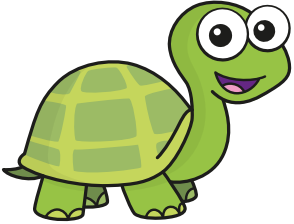
- Dora** $\frac{14}{4} = 3\frac{2}{4}$ **Mo** $\frac{14}{4} = 3\frac{1}{2}$

Whose conversion of $\frac{14}{4}$ do you agree with?

Convert improper fractions to mixed numbers

Reasoning and problem solving

$\frac{28}{3}$ is less than $\frac{37}{5}$ because 28 is less than 37



Do you agree with Tiny?
Explain your answer.

No


Which is greater, $\frac{19}{3}$ or $\frac{25}{4}$?

Explain your answer.

$\frac{19}{3}$

At half-time in a netball match, all seven players and both reserves are given half an orange.

How many oranges are needed altogether?



5 (there will be half an orange left over)

Work out the missing numbers.

$$\frac{23}{4} = \square \frac{\square}{\square}$$

$$23 \div 4 = \text{_____ remainder } \text{_____}$$

What do you notice?

$5 \frac{3}{4}$

5 remainder 3

Convert mixed numbers to improper fractions

Notes and guidance

This small step builds on the previous step and relies on children's understanding of the whole.

Children convert from mixed numbers to improper fractions by identifying how many of the equal parts each whole is worth and using this to work out how many equal parts are needed for the integer part of the mixed number. They then add on the number of parts in the fractional part of the mixed number and finally write the answer as an improper fraction.

As in the previous step, cubes, bar models and other representations can be used to support children's understanding.

Things to look out for

- Children may not make connections between the denominator of a fraction and the number of equal parts needed to make one whole.
- Instead of using multiplication, children may count each of the individual parts, which is inefficient and likely to lead to errors.
- Children may have the misconception that a fraction is always “part of a whole” and not realise that an improper fraction is a fraction that is greater than 1

Key questions

- How many _____ are there in one whole?
- How many _____ are there in _____ wholes?
- How many _____ are there altogether in the mixed number? How can you write this as an improper fraction?
- How many cubes do you need to represent the mixed number? How many cubes do you need for each whole? How many more cubes do you need? How many cubes do you need altogether?

Possible sentence stems

- Each whole can be split into _____
- The wholes are equal to _____ altogether.
- There are another _____ so the mixed number is _____ as an improper fraction.

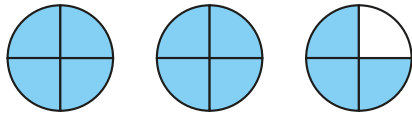
National Curriculum links

- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Convert mixed numbers to improper fractions

Key learning

- Each circle represents one whole.



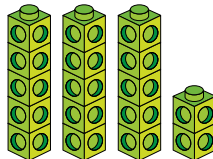
- ▶ What mixed number does the diagram show?
- ▶ What improper fraction does the diagram show?

- Each bar model represents one whole.



- ▶ What mixed number does the bar model show?
- ▶ What improper fraction does the bar model show?

- Kim uses cubes to help convert $3\frac{2}{5}$ to an improper fraction.



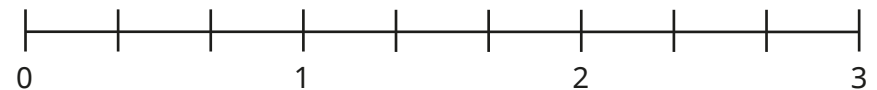
- ▶ Why has Kim built 3 towers of 5 cubes?
- ▶ Why are there only 2 cubes in the fourth tower?
- ▶ How many cubes has Kim used altogether?
- ▶ Write $3\frac{2}{5}$ as an improper fraction.

- Tom uses a bar model to convert $2\frac{3}{5}$ to an improper fraction. Complete Tom's workings.

	$2\frac{3}{5} =$ _____ wholes + _____ fifths
	2 wholes = _____ fifths
	_____ fifths + _____ fifths = _____ fifths
	$2\frac{3}{5} = \frac{\square}{5}$

Use Tom's method to convert $3\frac{2}{3}$, $2\frac{5}{6}$, $4\frac{3}{4}$ and $7\frac{1}{2}$ to improper fractions.

- Show $1\frac{2}{3}$ and $2\frac{1}{3}$ on the number line.



Write each mixed number as an improper fraction.


Use number lines to convert $3\frac{3}{4}$ and $3\frac{3}{5}$ to improper fractions.

- Which is greater, $4\frac{1}{4}$ or $\frac{21}{5}$?

Convert mixed numbers to improper fractions

Reasoning and problem solving

All the children in a class eat $\frac{1}{3}$ of a pizza at a party.



Altogether, they eat $8\frac{2}{3}$ pizzas.

How many children are there in the class?

26 children

Each episode of Dexter's favourite cartoon is $\frac{1}{4}$ hour long.

One day, Dexter watches his favourite cartoon for $2\frac{3}{4}$ hours without stopping!

How many episodes does Dexter watch?

How many episodes could Dexter watch in $4\frac{1}{2}$ hours?

11 episodes

18 episodes

How many different ways can you complete the statements?

$2\frac{\square}{8} = \frac{\square}{8}$

$2\frac{\square}{5} = \frac{\square}{5}$

seven possibilities

 $2\frac{1}{8} = \frac{17}{8}$ $2\frac{2}{8} = \frac{18}{8}$
 $2\frac{3}{8} = \frac{19}{8}$ $2\frac{4}{8} = \frac{20}{8}$
 $2\frac{5}{8} = \frac{21}{8}$ $2\frac{6}{8} = \frac{22}{8}$
 $2\frac{7}{8} = \frac{23}{8}$

four possibilities

 $2\frac{1}{5} = \frac{11}{5}$ $2\frac{2}{5} = \frac{12}{5}$
 $2\frac{3}{5} = \frac{13}{5}$ $2\frac{4}{5} = \frac{14}{5}$

Compare answers with a partner. What do you notice?

Compare fractions less than 1

Notes and guidance

Building on their knowledge of equivalent fractions, in this small step children compare fractions where the denominators are the same or where one denominator is a multiple of the other. They also compare fractions with the same numerator or by considering their position relative to one half.

Diagrams will help children to see which is the larger fraction and they should continue to use fraction walls and bar models until they are confident with the general rules.

The next step builds on this knowledge, with children ordering sets of fractions using the same techniques. They will also use equal denominators when adding and subtracting fractions and mixed numbers later in the block.

Things to look out for

- Children may mix up the meanings of the $>$ and $<$ symbols. In particular, they sometimes do not realise that statements such as $\frac{1}{2} > \frac{1}{3}$ and $\frac{1}{3} < \frac{1}{2}$ say the same thing even though the signs are different.
- When comparing fractions, children may focus solely on the numerators or the denominators.
- Children may incorrectly think that since $7 > 6$, $\frac{5}{7} > \frac{5}{6}$

Key questions

- If two fractions have the same denominator/numerator, how can you decide which is greater?
- How can you use equivalent fractions to help?
- How can you use a diagram to find equivalent fractions? Do the bars need to be the same size?

Possible sentence stems

- _____ is greater than one half, and _____ is less than one half, so _____ is greater than _____
- When two fractions have the same denominator, the one with the _____ numerator is the greater fraction.
- When two fractions have the same numerator, the one with the _____ denominator is the greater fraction.

National Curriculum links

- Compare and order fractions whose denominators are all multiples of the same number
- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

Compare fractions less than 1

Key learning

- Use diagrams to show that $\frac{4}{5} > \frac{3}{5}$
Explain how you can tell that $\frac{4}{5} > \frac{3}{5}$ without using a diagram.

- Use the bar models to compare the fractions.



$$\frac{2}{5} \bigcirc \frac{2}{3}$$



$$\frac{2}{3} \bigcirc \frac{2}{5}$$

- Identify the greater fraction in each pair.

▶ $\frac{3}{7}$ and $\frac{5}{7}$

▶ $\frac{9}{11}$ and $\frac{2}{11}$

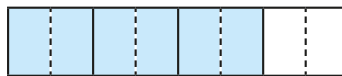
▶ $\frac{2}{3}$ and $\frac{2}{15}$

▶ $\frac{1}{8}$ and $\frac{1}{3}$

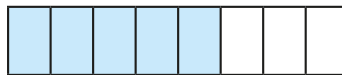
▶ $\frac{4}{15}$ and $\frac{4}{100}$

▶ $\frac{11}{20}$ and $\frac{19}{20}$

- Tommy uses bar models and equivalent fractions to compare $\frac{3}{4}$ and $\frac{5}{8}$



Which is the greater fraction, $\frac{3}{4}$ or $\frac{5}{8}$?



How do you know?

- Decide which is the greater fraction in each pair.

$$\frac{5}{6} \text{ and } \frac{2}{3}$$

$$\frac{2}{3} \text{ and } \frac{5}{9}$$

$$\frac{7}{16} \text{ and } \frac{3}{8}$$

- Esther and Aisha each have a chocolate bar.

Esther has eaten $\frac{3}{5}$ of her chocolate bar.

Aisha has eaten $\frac{8}{15}$ of her chocolate bar.

Who has eaten more of their chocolate bar?

- Write $<$, $>$ or $=$ to compare the fractions.

$$\frac{7}{8} \bigcirc \frac{3}{4}$$

$$\frac{6}{14} \bigcirc \frac{3}{7}$$

$$\frac{9}{10} \bigcirc \frac{9}{100}$$

$$\frac{1}{4} \bigcirc \frac{1}{40}$$

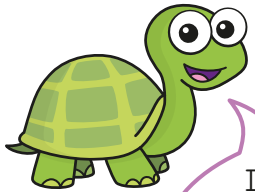
$$\frac{5}{9} \bigcirc \frac{9}{9}$$

$$\frac{1}{5} \bigcirc \frac{5}{20}$$

Compare fractions less than 1

Reasoning and problem solving

Tiny is comparing $\frac{7}{10}$ and $\frac{3}{8}$



I know that $\frac{5}{10}$ is equivalent to $\frac{1}{2}$ and $\frac{4}{8}$ is equivalent to $\frac{1}{2}$

How can Tiny use these facts to compare $\frac{7}{10}$ and $\frac{3}{8}$?

Use Tiny's method to compare the fractions.

$$\frac{17}{28} \quad \bigcirc \quad \frac{11}{32}$$

$$\frac{999}{2000} \quad \bigcirc \quad \frac{3007}{5000}$$

>
<

Annie and Jack are using a number line to work out which fraction is greater, $\frac{5}{6}$ or $\frac{2}{3}$

I looked to see how far away from zero each fraction was.



Annie



Jack

I looked to see how far away from 1 each fraction was.

Yes

Will Annie and Jack get the same answer?

Show working to support your answer.

Which method do you prefer?

Order fractions less than 1

Notes and guidance

In this small step, children build on their knowledge from the previous step to order a set of three or more fractions.

If equivalent fractions are needed, then one denominator will be a multiple of the other(s) so that conversions will not be complicated. Children will not need to compare fractions such as $\frac{2}{5}$ and $\frac{3}{7}$ until Year 6

Bar models, fraction walls and number lines will still be useful to help children to see the relative sizes of the fractions, especially when conversions are needed. Children should look at the set of fractions as a whole before deciding their approach, as comparing numerators could still be a better strategy for some sets of fractions.

Children can use other strategies covered in the previous step, such as considering the position of a fraction relative to 0, $\frac{1}{2}$ or 1 whole.

Things to look out for

- At first, children may need support to decide the best strategy when there are more than two fractions.
- Children may not look at both parts of the fractions when making their decisions about the order.

Key questions

- If a set of fractions all have the same denominator, how can you tell which is greatest?
- If a set of fractions all have the same numerator, how can you tell which is greatest?
- How can you use equivalent fractions to help?
- What are all the denominators/numerators multiples of? How can this help you find equivalent fractions?
- Which of the fractions are greater than $\frac{1}{2}$?

Possible sentence stems

- When fractions have the same denominator, the one with the _____ numerator is the greatest fraction.
- When fractions have the same numerator, the one with the _____ denominator is the greatest fraction.

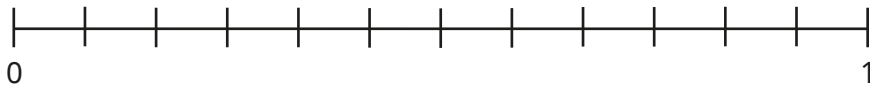
National Curriculum links

- Compare and order fractions whose denominators are all multiples of the same number
- Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths

Order fractions less than 1

Key learning

- Label $\frac{1}{12}$, $\frac{11}{12}$ and $\frac{7}{12}$ on the number line.



Write the three fractions in order of size, starting with the smallest.

- Complete the equivalent fractions.

$$\frac{1}{2} = \frac{\square}{12} \qquad \frac{3}{4} = \frac{\square}{12}$$

Use your answers to help you write these fractions in order of size, starting with the smallest.

$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{12}$	$\frac{1}{12}$
---------------	---------------	----------------	----------------

- Use equivalent fractions to write the fractions in ascending order.

$\frac{3}{5}$	$\frac{1}{2}$	$\frac{7}{10}$
---------------	---------------	----------------

- Order each set of fractions, from greatest to smallest.

▶ $\frac{3}{7}, \frac{3}{5}, \frac{3}{8}$ ▶ $\frac{2}{3}, \frac{5}{6}, \frac{7}{12}$ ▶ $\frac{1}{4}, \frac{2}{5}, \frac{3}{20}$

- Brett, Dani and Huan take the same spelling test.

Brett gets $\frac{3}{4}$ of the spellings correct.

Dani gets $\frac{3}{5}$ of the spellings correct.

Huan gets 19 of the 40 spellings correct.

- ▶ Who got the most spellings correct?
- ▶ Who got the least spellings correct?

- Order each set of fractions, from smallest to greatest.

$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{1}{7}$	$\frac{1}{3}$	$\frac{1}{9}$
---------------	---------------	---------------	----------------	---------------	---------------	---------------

$\frac{7}{9}$	$\frac{5}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{6}{9}$	$\frac{9}{9}$
---------------	---------------	---------------	---------------	---------------	---------------	---------------

$\frac{9}{20}$	$\frac{9}{10}$	$\frac{9}{100}$	$\frac{9}{1000}$	$\frac{9}{15}$	$\frac{9}{40}$
----------------	----------------	-----------------	------------------	----------------	----------------

- Order the fractions using the > symbol.

$\frac{5}{11}$	$\frac{5}{12}$	$\frac{6}{11}$
----------------	----------------	----------------

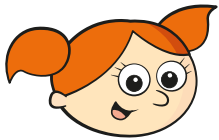
Order fractions less than 1

Reasoning and problem solving

Alex is ordering fractions.



I can put these fractions in order by comparing numerators.



$\frac{8}{15}$	$\frac{2}{7}$	$\frac{4}{11}$
----------------	---------------	----------------

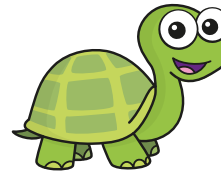
Explain how Alex can do this.

List the fractions in order, starting with the smallest.

8 and 4 are both multiples of 2, so Alex can rewrite the fractions such that all have a numerator of 8

As $\frac{8}{28} < \frac{8}{22} < \frac{8}{15}$,
the correct order is
 $\frac{2}{7} < \frac{4}{11} < \frac{8}{15}$

Tiny is ordering some fractions.



$\frac{1}{2} < \frac{2}{5} < \frac{3}{10} < \frac{7}{8}$
--

Explain the mistake Tiny has made.

Tiny has only looked at the numerators.

Fill in the boxes to make the statement true.



$$\frac{3}{8} < \frac{\square}{\square} < \frac{3}{4}$$

Complete the statement in two different ways.

Compare answers with a partner.



multiple possible answers, e.g.
 $\frac{4}{8} < \frac{5}{8} < \frac{3}{7} < \frac{3}{6} < \frac{3}{5}$

Compare and order fractions greater than 1

Notes and guidance

In this small step, children consolidate their knowledge from all the earlier steps in this block, using their skills in converting between forms to help compare and order fractions greater than 1

Children understand that if the number of wholes is different, they do not need to compare the fractional parts. When the number of wholes is equal, they compare denominators or numerators of the fractional parts. As with earlier steps, such comparisons will be limited to instances where the numerators or denominators are equal, or one denominator or numerator is a multiple of the other.

Again, diagrams will be helpful for students to see the comparative sizes of the numbers.

Things to look out for

- Children may need support to see the most efficient strategy, for example deciding whether any conversion is needed and/or whether to compare denominators or numerators.
- Errors may occur when converting between mixed numbers and improper fractions.

Key questions

- How can you represent the fractions?
- What does the number of wholes tell you about the overall sizes of the numbers?
- Do you need to make any conversions?
- How do you convert from an improper fraction/mixed number to a mixed number/improper fraction?
- How can you use your knowledge of multiples to help?

Possible sentence stems

- If the number of wholes is different, then the number with _____ wholes is greater.
- If the number of wholes is equal, then I need to compare the _____

National Curriculum links

- Compare and order fractions whose denominators are all multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Compare and order fractions greater than 1

Key learning

- Draw a diagram to show how you know that $3\frac{1}{3} > 2\frac{1}{2}$

- Write $<$ or $>$ to compare the numbers.

$$4\frac{1}{2} \bigcirc 3\frac{1}{2} \qquad 5\frac{1}{3} \bigcirc 4$$

$$2\frac{4}{5} \bigcirc 3\frac{1}{4} \qquad 3 \bigcirc 4\frac{1}{3}$$

- Put the numbers in order, starting with the smallest.

$$3\frac{1}{3} \quad 5\frac{1}{2} \quad 2\frac{3}{4} \quad 4\frac{1}{5} \quad 1\frac{9}{10}$$

- Explain how the diagrams show that $2\frac{1}{3} < 2\frac{1}{2}$

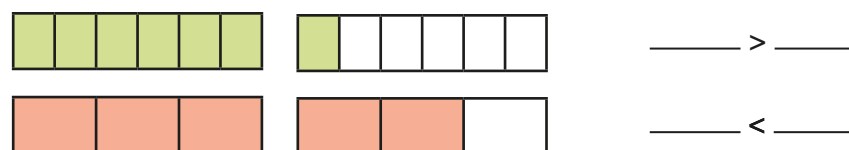


- Write $<$ or $>$ to compare the numbers.

$$4\frac{1}{2} \bigcirc 4\frac{1}{5} \qquad 2\frac{4}{5} \bigcirc 2\frac{4}{7}$$

$$5\frac{3}{4} \bigcirc 5\frac{1}{4} \qquad 3\frac{5}{8} \bigcirc 3\frac{7}{8}$$

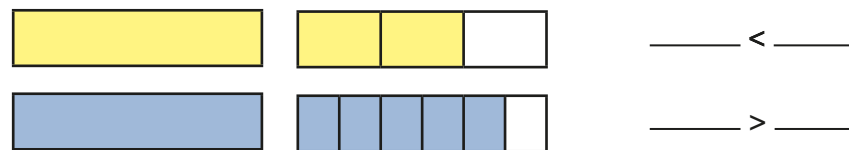
- Use the bar models to compare $\frac{7}{6}$ and $\frac{5}{3}$



In each pair of fractions, decide which fraction is greater.

- $\frac{5}{2}$ and $\frac{9}{4}$
- $\frac{11}{6}$ and $\frac{5}{3}$
- $\frac{9}{4}$ and $\frac{17}{8}$

- Use the bar models to compare $1\frac{2}{3}$ and $1\frac{5}{6}$



In each pair of mixed numbers, decide which is greater.

- $1\frac{5}{8}$ and $1\frac{1}{2}$
- $2\frac{3}{7}$ and $2\frac{9}{14}$
- $3\frac{3}{4}$ and $3\frac{7}{8}$

- Use common denominators to put each set of numbers in order, starting with the smallest.

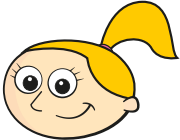


Compare and order fractions greater than 1

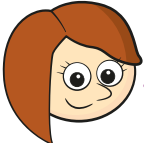
Reasoning and problem solving

Eva and Rosie each have two identical pizzas.

I have cut each pizza into 6 equal pieces and eaten 8 pieces.



Eva



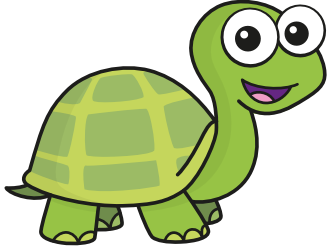
Rosie

I have cut each pizza into 9 equal pieces and eaten 15 pieces.

Who has eaten more pizza?
Explain how you know.

Rosie

I can find mixed numbers and improper fractions that make the statement correct.



$$2\frac{3}{4} < \square < \frac{10}{3}$$

What mixed numbers and improper fractions can Tiny find?
Compare answers with a partner.

multiple possible answers, e.g.

$$2\frac{7}{8} = \frac{21}{8}$$

$$2\frac{9}{10} = \frac{29}{10}$$

$$3\frac{1}{10} = \frac{31}{10}$$

$$3\frac{1}{5} = \frac{16}{5}$$

$$3\frac{1}{4} = \frac{13}{4}$$

Add and subtract fractions with the same denominator

Notes and guidance

In this small step, children add and subtract fractions with the same denominator. For adding, this will include adding three or more fractions as well as pairs of fractions.

Children need to understand that when the denominators are the same, they only need to add or subtract the numerators. Bar models are a useful way of representing both addition and subtraction of fractions and are easier to work with than circles, as they are easier to draw and easier to split into equal parts.

Now that children are comfortable working with improper fractions, these are incorporated into both the questions and answers of this small step. For some questions, answers could be written in a simplified form, but this is not the focus of the step.

Things to look out for

- A common error is to add or subtract both the numerators and the denominators, for example $\frac{3}{4} + \frac{3}{4} = \frac{6}{8}$ or $\frac{5}{6} - \frac{1}{6} = \frac{4}{0}$
- Children may not look at the symbols in the calculations and, for example, add the fractions when the intended calculation is a subtraction.

Key questions

- How can you represent this calculation using a bar model?
- Will you need more than one bar? How do you know?
- How many parts do you split the bar(s) into?
- What could you do if the answer is an improper fraction?
- What type of calculation is this?
- When adding/subtracting fractions with the same denominators, what will the denominator of the answer be? Why?

Possible sentence stems

- _____ fifths add/subtract _____ fifths is _____ fifths.
- When adding/subtracting fractions with the same denominators, I just add/subtract the _____

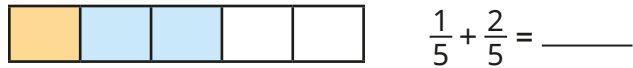
National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Add and subtract fractions with the same denominator

Key learning

- Use the bar model to complete the calculation.



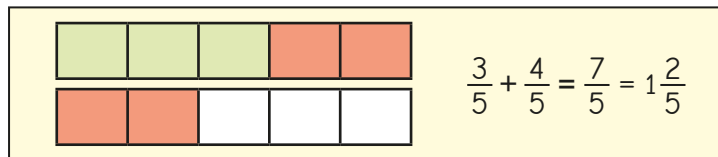
- Use the bar model to help you work out $\frac{9}{5} - \frac{6}{5}$



- Work out the calculations.

$\triangleright \frac{4}{9} + \frac{1}{9}$ $\triangleright \frac{3}{7} + \frac{2}{7}$ $\triangleright \frac{4}{9} + \frac{1}{9} + \frac{2}{9}$
 $\triangleright \frac{7}{9} - \frac{5}{9}$ $\triangleright \frac{1}{8} + \frac{3}{8} + \frac{3}{8}$ $\triangleright \frac{6}{7} - \frac{4}{7}$

- Teddy uses a bar model to work out $\frac{3}{5} + \frac{4}{5}$



Use bar models to find the totals.

$$\frac{2}{3} + \frac{2}{3}$$

$$\frac{3}{4} + \frac{3}{4} + \frac{1}{4}$$

$$\frac{6}{5} + \frac{4}{5}$$

- Work out the missing numbers.

$\triangleright \frac{3}{7} + \frac{\square}{7} = \frac{9}{7}$ $\triangleright \frac{3}{11} + \frac{\square}{11} = \frac{15}{11}$ $\triangleright \frac{15}{8} - \frac{\square}{8} = 1$
 $\triangleright \frac{\square}{5} - \frac{3}{5} = \frac{4}{5}$ $\triangleright \frac{4}{5} + \frac{\square}{5} = 2$ $\triangleright \frac{10}{3} - \frac{\square}{3} = 2$

- A flag is made from different-coloured parts.

- $\frac{2}{15}$ of the flag is blue.
- $\frac{7}{15}$ of the flag is red.
- $\frac{1}{15}$ of the flag is green.

- What fraction of the flag is blue, red or green?
The rest of the flag is white.
- What fraction of the flag is white?

- Work out the missing numbers.

$\triangleright \frac{3}{7} + \frac{5}{7} = \frac{\square}{\square} + \frac{4}{7}$ $\triangleright \frac{9}{5} - \frac{5}{5} = \frac{6}{5} - \frac{\square}{\square}$ $\triangleright \frac{2}{3} + \frac{\square}{\square} = \frac{11}{3} - \frac{4}{3}$

Add and subtract fractions with the same denominator

Reasoning and problem solving

Find as many ways as you can to make the statement correct.

$$\frac{5}{9} + \frac{\square}{9} = \frac{8}{9} + \frac{\square}{9}$$

Compare answers with a partner.



multiple possible answers, e.g.

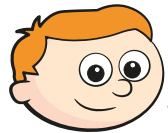
$$\frac{5}{9} + \frac{4}{9} = \frac{8}{9} + \frac{1}{9}$$

$$\frac{5}{9} + \frac{5}{9} = \frac{8}{9} + \frac{2}{9}$$

$$\frac{5}{9} + \frac{6}{9} = \frac{8}{9} + \frac{3}{9}$$



Ron adds three fractions with the same denominator.



The answer is $\frac{17}{18}$

What three fractions could he have added?



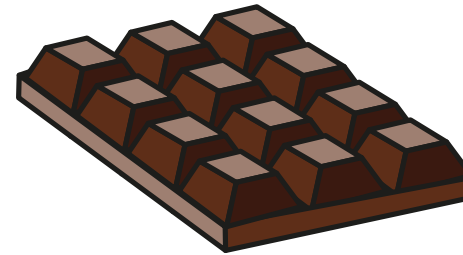
multiple possible answers, e.g.

$$\frac{1}{18} + \frac{3}{18} + \frac{13}{18}$$

$$\frac{1}{18} + \frac{5}{18} + \frac{11}{18}$$

$$\frac{6}{18} + \frac{3}{18} + \frac{8}{18}$$

A chocolate bar has 12 equal pieces.



- Amir eats $\frac{5}{12}$ more of the bar than Whitney.
- There is $\frac{1}{12}$ of the bar left.

What fraction of the bar does Amir eat?

What fraction of the bar does Whitney eat?

$$\frac{8}{12}$$

$$\frac{3}{12}$$

Add fractions within 1

Notes and guidance

In this small step, children add two or three fractions with different denominators. The fractions are such that one denominator is a multiple of another and the total remains within 1. Children may be familiar with some simple common additions, such as $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$, and this could be a good example on which to build.

Children can use pictorial representations to convert one of the fractions so they have a common denominator and to support the addition. If they write their workings alongside the pictorial representations, this will help them to make the links.

Things to look out for

- Children may not realise the need to make the denominators equal before adding.
- Children may add both the numerators and the denominators, for example $\frac{1}{2} + \frac{1}{4} = \frac{2}{6}$
- Children may make mistakes when finding equivalent fractions.
- Children may think that the only way to find a common denominator is to find the product of the denominators in the question.

Key questions

- Do the fractions have the same denominator?
- What does it mean for two fractions to be equivalent?
- How can you tell when two fractions are equivalent?
- Why do the denominators need to be the same?
- How can you find a common denominator?
- How many of the fractions do you need to convert?
- Now the denominators are the same, how do you add the fractions?

Possible sentence stems

- Fractions must have the same _____ before you can add them.
- The denominator has been multiplied by _____, so the numerator needs to be multiplied by _____ for the fractions to be equivalent.

National Curriculum links

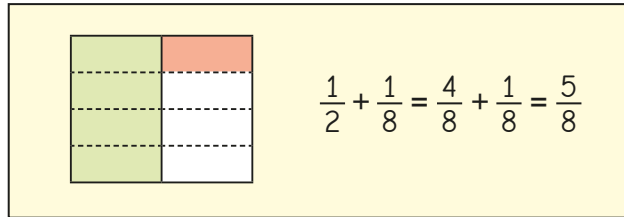
- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number

Add fractions within 1

Key learning

- Scott is working out $\frac{1}{2} + \frac{1}{8}$

He uses a diagram to represent the sum.

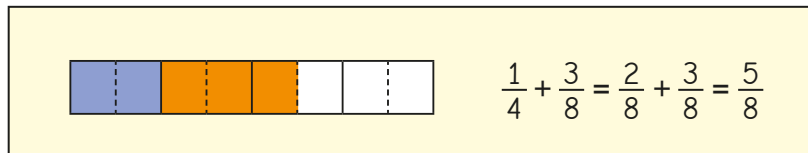


Use Scott's method to work out the additions.

- ▶ $\frac{1}{2} + \frac{3}{8}$
- ▶ $\frac{1}{4} + \frac{5}{8}$
- ▶ $\frac{7}{10} + \frac{1}{5}$

- Draw a diagram to show that $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

- Rosie uses a bar model to work out $\frac{1}{4} + \frac{3}{8}$



Use a bar model to work out the additions.

- ▶ $\frac{1}{6} + \frac{5}{12}$
- ▶ $\frac{2}{9} + \frac{1}{3}$
- ▶ $\frac{1}{3} + \frac{4}{15}$

- What common denominator would you use to work out each addition?

$\frac{1}{2} + \frac{1}{6}$

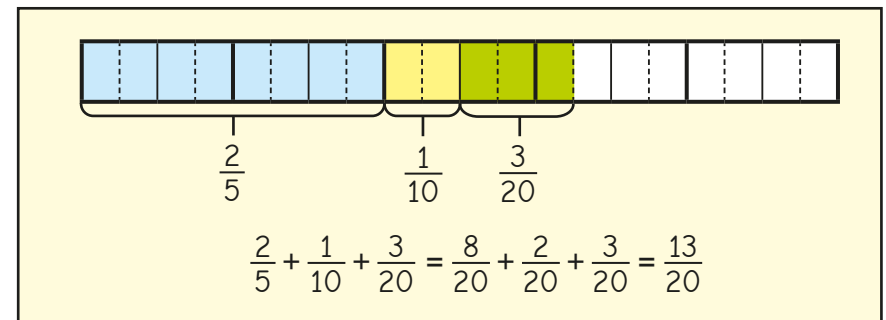
$\frac{1}{4} + \frac{5}{8}$

$\frac{3}{10} + \frac{2}{5}$

$\frac{1}{6} + \frac{1}{3}$

Work out the additions.

- Nijah uses a bar model to work out $\frac{2}{5} + \frac{1}{10} + \frac{3}{20}$



Work out the additions.

$\frac{1}{4} + \frac{3}{8} + \frac{5}{16}$

$\frac{1}{2} + \frac{1}{6} + \frac{1}{12}$

$\frac{1}{3} + \frac{2}{9} + \frac{5}{18}$

- Work out the missing numbers.

- ▶ $\frac{1}{5} + \square + \frac{8}{20} = 1$
- ▶ $\frac{1}{5} + \square + \frac{1}{30} = 1$

Add fractions within 1

Reasoning and problem solving

Work out the missing numbers.

$$\frac{5}{16} + \frac{\square}{8} = \frac{15}{16}$$

5, 3

$$\frac{\square}{20} + \frac{7}{10} = \frac{17}{20}$$

Tommy adds three fractions with different denominators.



The answer is $\frac{17}{18}$

What three fractions could he have added?

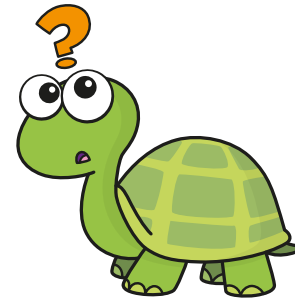
multiple possible answers, e.g.

$$\frac{1}{18} + \frac{2}{9} + \frac{2}{3}$$

$$\frac{3}{18} + \frac{4}{9} + \frac{1}{3}$$

Tiny is trying to work out this addition.

$$\frac{3}{5} + \frac{1}{10} + \frac{3}{20}$$



$\frac{17}{20}$

$$\frac{3}{5} + \frac{1}{10} + \frac{3}{20} = \frac{7}{35}$$

What mistake has Tiny made?

Find the correct total.

Add fractions with total greater than 1

Notes and guidance

In this small step, children continue to add fractions where one denominator is a multiple of the other, but progress to additions where the total is greater than 1. Their answers will be improper fractions that they should convert to mixed numbers using the skills they have learnt in earlier steps.

Children continue to represent the addition of fractions using pictorial or concrete representations to make sense of both the methods and the answers.

They need to be clear what represents the whole in each case.

Things to look out for

- Children may forget to find a common denominator before adding the fractions.
- Children may add the denominators as well as the numerators.
- Children may misinterpret their diagrams, not realising that the answer is more than one whole, for example interpreting this diagram as $\frac{5}{6}$ instead of $1\frac{2}{3}$



Key questions

- Do the fractions have the same denominator?
- How can you find a common denominator?
- How many of the fractions do you need to convert?
- Now the denominators are the same, how do you add the fractions?
- How can you tell the answer is greater than one whole?
- How can you convert the answer to a mixed number?

Possible sentence stems

- I am going to make all of the denominators _____
- I can regroup _____ to make _____ wholes with _____ parts left over. As a mixed number, this is _____ and _____

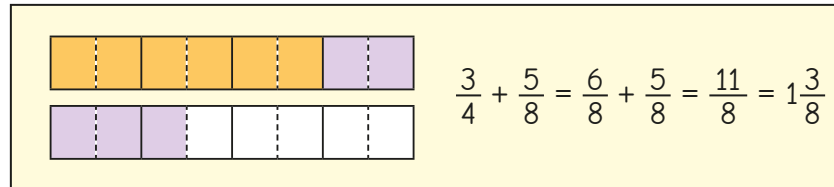
National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Add fractions with total greater than 1

Key learning

- Dora uses a bar model to add $\frac{3}{4}$ and $\frac{5}{8}$

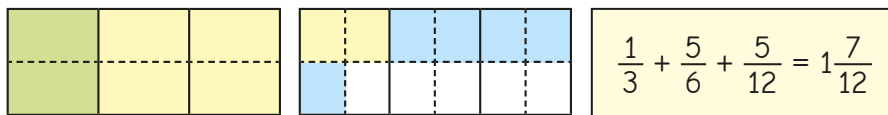


Use Dora's method to work out the additions.

Give your answers as mixed numbers.

▶ $\frac{2}{3} + \frac{5}{12}$ ▶ $\frac{5}{8} + \frac{11}{16}$ ▶ $\frac{7}{10} + \frac{3}{5}$

- Here is Mo's method for adding three fractions with different denominators.



Explain each step of the calculation.

Add the sets of fractions, giving your answers as mixed numbers.

▶ $\frac{2}{3} + \frac{1}{6} + \frac{7}{12}$ ▶ $\frac{1}{4} + \frac{7}{8} + \frac{3}{16}$ ▶ $\frac{1}{2} + \frac{5}{6} + \frac{5}{12}$

- Use the bar model to work out $\frac{3}{4} + \frac{3}{8} + \frac{1}{2}$

Give your answer as a mixed number.



Draw your own bar models to add the fractions, giving your answers as mixed numbers.

$$\frac{5}{12} + \frac{1}{6} + \frac{1}{2}$$

$$\frac{11}{20} + \frac{3}{5} + \frac{1}{10}$$

$$\frac{3}{4} + \frac{5}{12} + \frac{1}{2}$$

- What common denominator would you use to work out these additions?

$$\frac{1}{2} + \frac{5}{6} + \frac{7}{12}$$

$$\frac{2}{5} + \frac{7}{20} + \frac{9}{10}$$

$$\frac{4}{15} + \frac{2}{3} + \frac{11}{30}$$

Work out the additions.

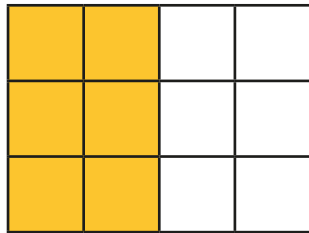
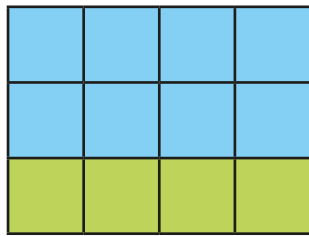
- Work out the missing numbers.

▶ $\frac{7}{10} + \square = 1\frac{3}{10}$ ▶ $\frac{3}{4} + \frac{7}{8} + \square = 2$ ▶ $3\frac{1}{12} = \square + \frac{2}{3} + \frac{5}{6}$

Add fractions with total greater than 1

Reasoning and problem solving

Kim uses the diagram to add three fractions.



What could her fractions be?
How many different combinations can you find?



multiple possible answers, e.g.

$$\frac{2}{3} + \frac{4}{12} + \frac{1}{2}$$

$$\frac{\square}{\square} + \frac{\square}{\square} + \frac{\square}{\square} = 2\frac{1}{8}$$



The sum of three fractions is $2\frac{1}{8}$

Use the clues to work out the three fractions.

- All the fractions have different denominators.
- None of the fractions are smaller than one half.
- None of the fractions are improper.
- All three denominators are factors of 8

$$\frac{1}{2}, \frac{3}{4}, \frac{7}{8}$$

Investigate different sets of fractions that have the same total but satisfy fewer of the clues.



Add to a mixed number

Notes and guidance

In this small step, children add either a whole number part or a fractional part to a mixed number as a precursor to adding two mixed numbers in the next step.

The key point is that children remember that a mixed number such as $3\frac{1}{2}$ can be partitioned into $3 + \frac{1}{2}$ and then they can add to the required part before recombining. The expectation is that, provided the sum of fractional parts does not cross a whole, these additions will generally be done mentally. Pictorial support may still be useful, initially.

Crossing the whole will be included towards the end of this step and will feature more prominently in the next step.

Things to look out for

- Children may need to be reminded how to partition mixed numbers.
- Children may omit one or more of the numbers being added in a calculation and not recombine all the parts into the correct mixed number at the end.
- When totals cross a whole, children need to be confident converting improper fractions. Recombining will require careful recording to prevent slips.

Key questions

- How can you partition a mixed number?
- How can the addition be written so that similar parts are next to each other?
- How can the parts be combined to produce a mixed number?
- Do you need to combine whole numbers or fractions?
- Why can you swap the order of the numbers in an addition?

Possible sentence stems

- A mixed number can be partitioned into a _____ part and a _____ part.
- The fractional part of the answer is an _____, so needs converting to a _____

National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

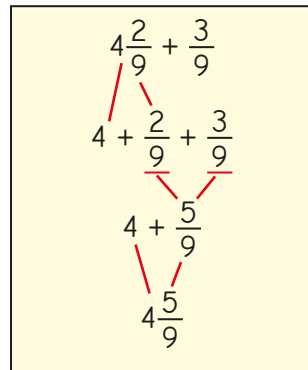
Add to a mixed number

Key learning

- Tom adds a fraction to a mixed number by adding the fractions separately and then adding the wholes.

Use Tom's method to work out the additions.

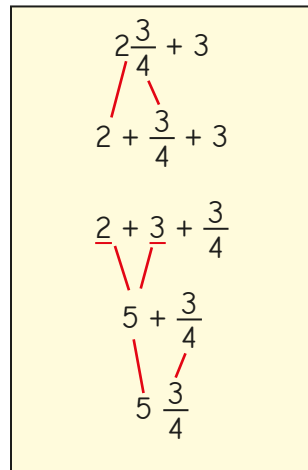
$3\frac{1}{5} + \frac{3}{5}$	$4\frac{1}{3} + \frac{1}{3}$
$\frac{2}{7} + 3\frac{4}{7}$	$\frac{2}{9} + 3\frac{5}{9}$



- Here is Esther's method for working out $2\frac{3}{4} + 3$

Use Esther's method to work out the additions.

$3\frac{1}{2} + 2$	$6 + 2\frac{1}{5}$
$5\frac{3}{7} + 2$	$3 + 2\frac{4}{5}$



- Aisha thinks $4\frac{3}{8} + \frac{1}{8} = 4\frac{4}{8}$
- Dani thinks $4\frac{3}{8} + \frac{1}{8} = 4\frac{1}{2}$

Who do you agree with?

- Work out the totals.

$3\frac{2}{9} + \frac{4}{9}$	$2\frac{5}{8} + \frac{1}{8}$	$\frac{5}{12} + 4\frac{5}{12}$
------------------------------	------------------------------	--------------------------------

- Kim uses equivalent fractions to work out $4\frac{1}{3} + \frac{1}{6}$

$$4\frac{1}{3} + \frac{1}{6} = 4 + \frac{1}{3} + \frac{1}{6}$$

$$= 4 + \frac{2}{6} + \frac{1}{6}$$

$$= 4\frac{3}{6}$$

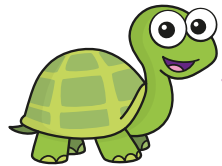
$$= 4\frac{1}{2}$$

Work out the totals.

$3\frac{2}{5} + \frac{3}{10}$	$2\frac{7}{12} + \frac{1}{6}$	$\frac{3}{8} + 3\frac{3}{16}$	$1\frac{2}{5} + \frac{4}{15}$
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Add to a mixed number

Reasoning and problem solving



$$5\frac{2}{3} + \frac{1}{3} = 5\frac{3}{3}$$

How can you improve on Tiny's answer?

Here are some additions.

$\frac{5}{12} + 2\frac{7}{12}$	$6\frac{3}{5} + \frac{2}{5}$	$6\frac{3}{5} + \frac{3}{5}$
$\frac{4}{7} + 2\frac{5}{7}$	$3\frac{7}{8} + \frac{5}{8}$	

Work out the totals.

Think carefully about how you give your answers.

3, 7, $7\frac{1}{5}$
 $3\frac{2}{7}$, $4\frac{1}{2}$

Here are some fraction cards.

$\frac{4}{5}$	$\frac{2}{3}$	$\frac{1}{4}$
---------------	---------------	---------------

Decide which fraction card belongs in which calculation.

$$4\frac{7}{9} = 4\frac{1}{9} + \square$$

$$3\frac{3}{8} + \square = 3\frac{5}{8}$$

$$\frac{43}{10} = 3\frac{1}{2} + \square$$

$\frac{2}{3}$
 $\frac{1}{4}$
 $\frac{4}{5}$

What could the values of A and B be?

$$A\frac{5}{12} + \frac{B}{4} = 5\frac{1}{6}$$

Compare answers with a partner.

multiple possible answers, e.g.
 A = 4 and B = 3

Add two mixed numbers

Notes and guidance

Building on the previous step, children add two mixed numbers by adding the wholes and fractional parts separately. This is usually the most efficient method of adding two mixed numbers, but converting to improper fractions and adding them is included as an alternative. Examples are included where children need to use equivalent fractions and where answers can be simplified, although simplifying answers is not the priority here.

Children can still draw models to represent adding fractions, particularly if these are useful for pairs of fractions with different denominators. The cognitive load is significant when finding solutions to these multi-step problems, so providing scaffolding/partially started solutions may be useful.

Things to look out for

- Children may make errors in the partitioning or recombining of the integer and fractional parts.
- Arithmetical errors may occur when converting to improper fractions with larger numbers.
- Where the fractional parts cross the whole, children may not interpret this correctly, either leaving their answer as a whole and improper fraction, or converting but not adding 1 to the integer.

Key questions

- How can you partition the mixed numbers?
- How can the addition be rewritten to make it easier?
- Do you need to combine whole numbers, fractions or both?
- Are there any improper fractions in the answer?
What can you do about this?
- How do you change a mixed number into an improper fraction?
- In this question, is it easier to deal with mixed numbers or to use improper fractions? Why?

Possible sentence stems

- The mixed numbers can be partitioned into a _____ part and a _____ part.

National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Add two mixed numbers

Key learning

- Use bar models to show that $2\frac{2}{5} + 3\frac{1}{5} = 5\frac{3}{5}$
- Annie adds two mixed numbers by adding the wholes first and then adding the fractions.

$$2\frac{3}{5} + 4\frac{1}{5} = 6 + \frac{4}{5} = 6\frac{4}{5}$$

Use Annie's method to find the totals.

$$\triangleright 3\frac{4}{9} + 2\frac{1}{9} \quad \triangleright 1\frac{2}{7} + 4\frac{3}{7} \quad \triangleright 5\frac{11}{15} + 3\frac{2}{15}$$

- Use bar models to show that $2\frac{1}{2} + 3\frac{1}{4} = 5\frac{3}{4}$
- Filip uses equivalent fractions to add $1\frac{1}{3}$ and $2\frac{1}{6}$

$$1\frac{1}{3} + 2\frac{1}{6} = 1 + 2 + \frac{1}{3} + \frac{1}{6} = 3 + \frac{2}{6} + \frac{1}{6} = 3\frac{3}{6} = 3\frac{1}{2}$$

Use Filip's method to find the totals.

$$\triangleright 3\frac{1}{4} + 2\frac{3}{8} \quad \triangleright 4\frac{1}{9} + 3\frac{2}{3} \quad \triangleright 2\frac{5}{12} + 2\frac{1}{3}$$

- Jack adds $1\frac{3}{4}$ to $2\frac{1}{8}$ by converting them to improper fractions.

$$1\frac{3}{4} + 2\frac{1}{8} = \frac{7}{4} + \frac{17}{8} = \frac{14}{8} + \frac{17}{8} = \frac{31}{8} = 3\frac{7}{8}$$

Use Jack's method to find the totals.

$$\triangleright 1\frac{1}{4} + 2\frac{5}{12} \quad \triangleright 2\frac{1}{9} + 1\frac{1}{3} \quad \triangleright 2\frac{1}{6} + 2\frac{2}{3}$$

Would you use Jack's method or a different method to find the answer to $6\frac{3}{7} + 9\frac{3}{14}$?

- Alex adds $5\frac{4}{5}$ and $4\frac{3}{5}$ by adding the wholes first and then adding the fractions.

$$5\frac{4}{5} + 4\frac{3}{5} = 9 + \frac{7}{5} = 9\frac{7}{5}$$

How can Alex's answer be improved?

Work out the additions.

$$\triangleright 4\frac{7}{9} + 2\frac{1}{3} \quad \triangleright 2\frac{5}{6} + 1\frac{1}{3} \quad \triangleright \frac{15}{8} + 2\frac{1}{4}$$

Add two mixed numbers

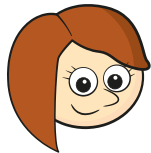
Reasoning and problem solving

Amir and Rosie measure how much water they drink one day.

I drank $2\frac{1}{4}$ litres.



Amir



Rosie

I drank $2\frac{5}{12}$ litres.

How much water do they drink altogether that day?

How many different ways can you find to work out the answer?

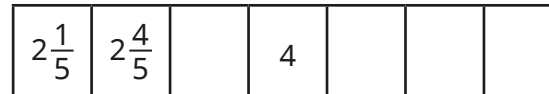
Which method do you think is most efficient?

Explain your answer.

$4\frac{2}{3}$ litres

Discuss preferred methods (e.g. wholes and fractions, improper fractions, diagrams) as a class.

Here are some sequences on number tracks.



Fill in the missing numbers.

$3\frac{2}{5}, 4\frac{3}{5}, 5\frac{1}{5}, 5\frac{4}{5}$
 $2\frac{1}{7}, 2\frac{6}{7}, 3\frac{4}{7}$

Complete the addition.

$$3\frac{2}{5} + \square = \frac{81}{10}$$

$4\frac{7}{10}$

Subtract fractions

Notes and guidance

Children subtracted fractions with the same denominators earlier in this block. In this small step, they now move on to subtract fractions where one denominator is a multiple of the other, using the same skills they learned for adding fractions of this type.

Both proper and improper fractions are included, but this step does not include mixed numbers, conversions and crossing the whole; these will follow in subsequent steps. It is useful to consider subtraction in all its forms: partitioning, reduction and finding the difference. Pictorial representations such as bar models and number lines will help support understanding.

Things to look out for

- Children may not realise the need to make the denominators equal before subtracting.
- Children may subtract both the numerators and the denominators, for example $\frac{7}{8} - \frac{1}{3} = \frac{6}{5}$
- Errors may occur when finding equivalent fractions or misreading the question and adding instead of subtracting.
- Children may think that the only way to find a common denominator is to multiply the two denominators.

Key questions

- Do the fractions have the same denominator?
- When are two fractions equivalent?
- How can you find a common denominator?
- How many of the fractions do you need to convert?
- Now the denominators are the same, how do you subtract the fractions?
- How can you represent the problem using a diagram?

Possible sentence stems

- Fractions must have the same _____ before they can be subtracted.
- The denominator has been multiplied by _____, so the numerator needs to be multiplied by _____ for the fractions to be equivalent.

National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number

Subtract fractions

Key learning

- Eva is working out $\frac{1}{3} - \frac{1}{15}$

$$\frac{1}{3} - \frac{1}{15} = \frac{5}{15} - \frac{1}{15} = \frac{4}{15}$$

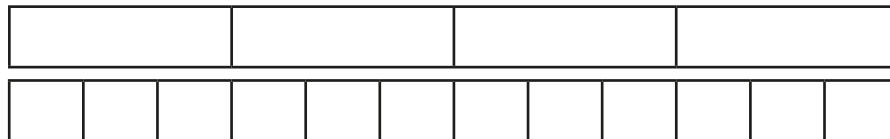
Explain each step in her calculation.

Use Eva's method to work out the subtractions.

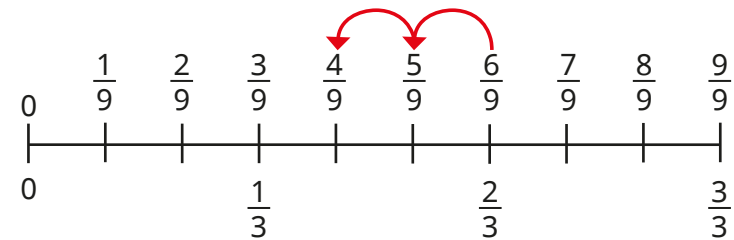
$\frac{5}{6} - \frac{2}{3}$	$\frac{7}{8} - \frac{5}{16}$	$\frac{1}{2} - \frac{1}{10}$
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- Huan and Whitney both started with the same size chocolate bar. Huan has $\frac{3}{4}$ of his chocolate bar left and Whitney has $\frac{5}{12}$ of her chocolate bar left.

How much more chocolate does Huan have than Whitney?



- The number line shows that $\frac{2}{3} - \frac{2}{9} = \frac{4}{9}$



Work out the subtractions.

$\frac{2}{3} - \frac{5}{9}$	$\frac{7}{9} - \frac{4}{9}$	$\frac{8}{9} - \frac{1}{3}$
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- Find the difference between each pair of fractions.

$\frac{5}{12}$ and $\frac{3}{4}$	$\frac{3}{5}$ and $\frac{19}{15}$	$\frac{20}{9}$ and $\frac{4}{3}$
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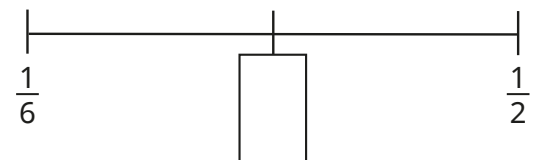
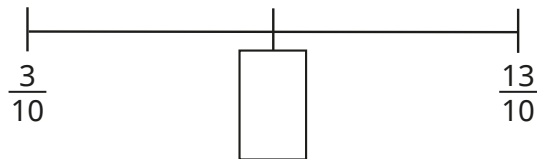
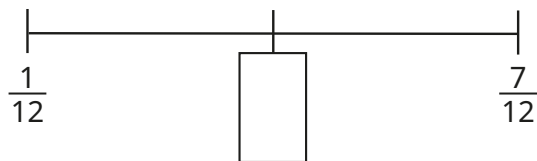
- Work out the subtractions.

$\frac{3}{4} - \frac{5}{16}$	$\frac{7}{3} - \frac{8}{9}$	$\frac{11}{12} - \frac{2}{3}$
$\frac{17}{20} - \frac{4}{5}$	$\frac{12}{5} - \frac{7}{10}$	$\frac{14}{14} - \frac{11}{14}$

Subtract fractions

Reasoning and problem solving

Find the value of the midpoint of each number line.

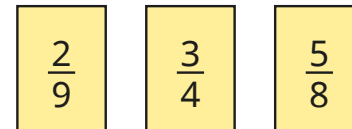
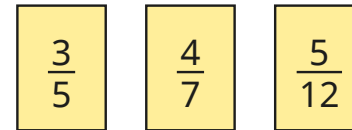


$\frac{4}{12}$ (or $\frac{1}{3}$)

$\frac{8}{10}$ (or $\frac{4}{5}$)

$\frac{2}{6}$ (or $\frac{1}{3}$)

Subtract each fraction from one whole.



$\frac{2}{5}$, $\frac{3}{7}$, $\frac{7}{12}$

$\frac{7}{9}$, $\frac{1}{4}$, $\frac{3}{8}$

What connections can you see between the fractions and your answers?

Two fractions have a difference of one half.

What could the fractions be?

Compare answers with a partner.

multiple possible answers, e.g.

$\frac{7}{4}$ and $\frac{5}{4}$

Subtract from a mixed number

Notes and guidance

In a previous step, children added to a mixed number as a prerequisite for adding mixed numbers; in this small step, they look at a similar process for subtracting. Children subtract either a whole number part or a fractional part from a mixed number. Crossing the whole is not included, as this is the focus of the next step.

Encourage children who need support to continue to use concrete resources and pictorial representations to make sense of the methods. This step provides more opportunities to develop their understanding of equivalent fractions, as some of the denominators are multiples of the other denominator in the calculation. Although some answers could be simplified, this is not the focus of the step.

Things to look out for

- Children may need support in partitioning mixed numbers.
- Children may overcomplicate the calculations by converting mixed numbers to improper fractions, which will not be necessary for subtractions of this type.
- Children may not pay attention to the calculation and add instead of subtract.

Key questions

- How can you partition a mixed number?
- Can the subtraction be written in a different form to make it easier?
- If the denominators are different, what do you need to do?
- How can the parts be combined to produce a mixed number?
- Do you need to combine whole numbers or fractions?
- Can you change the order of the numbers in a subtraction?

Possible sentence stems

- A mixed number can be partitioned into a _____ part and a _____ part.
- The difference between the wholes is _____
- The difference between the fractions is _____

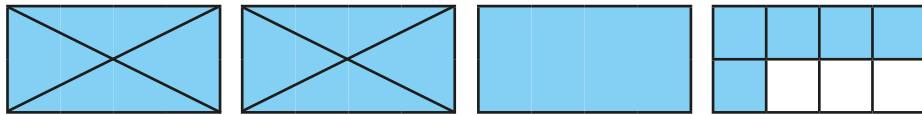
National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

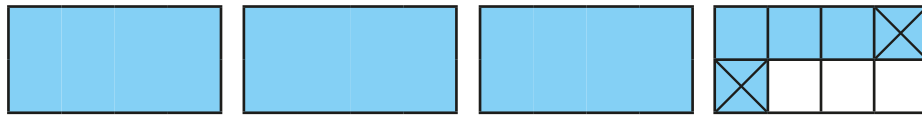
Subtract from a mixed number

Key learning

- Explain how the diagram shows $3\frac{5}{8} - 2 = 1\frac{5}{8}$



What calculation does this diagram show?



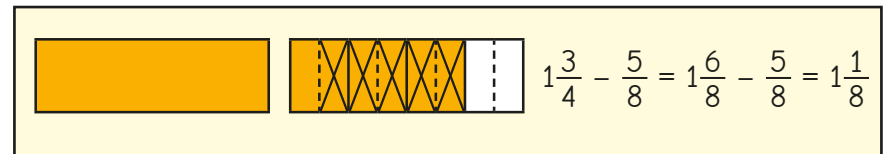
- Work out the subtractions.

▶ $3\frac{4}{5} - 1$ ▶ $3\frac{4}{5} - 2$ ▶ $6\frac{3}{5} - 4$
 ▶ $3\frac{4}{5} - \frac{1}{5}$ ▶ $3\frac{4}{5} - \frac{2}{5}$ ▶ $3\frac{4}{5} - \frac{3}{5}$

- Work out the missing numbers.

▶ $4\frac{3}{4} - \underline{\hspace{2cm}} = 1\frac{3}{4}$ ▶ $3\frac{5}{7} - \underline{\hspace{2cm}} = 3\frac{1}{7}$ ▶ $\underline{\hspace{2cm}} - \frac{2}{9} = 6\frac{5}{9}$

- Here is Ron's method for working out $1\frac{3}{4} - \frac{5}{8}$

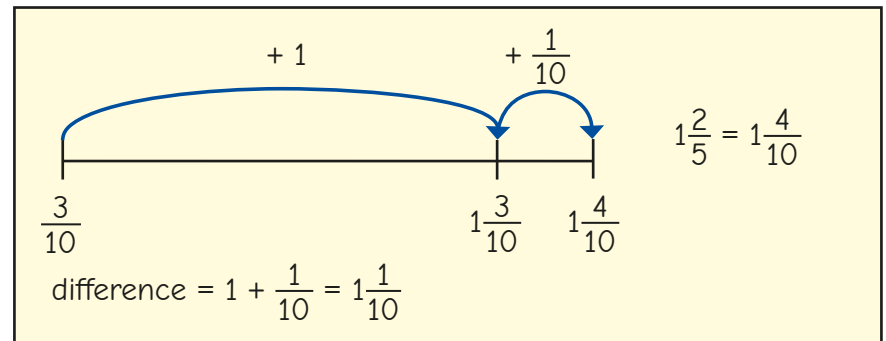


Explain the steps in Ron's method.

Use Ron's method to work out the subtractions.

▶ $2\frac{3}{5} - \frac{3}{10}$ ▶ $2\frac{2}{3} - \frac{1}{6}$ ▶ $1\frac{11}{12} - \frac{5}{6}$

- Kim uses a number line to find the difference between $1\frac{2}{5}$ and $\frac{3}{10}$



Find the difference between each pair of fractions.

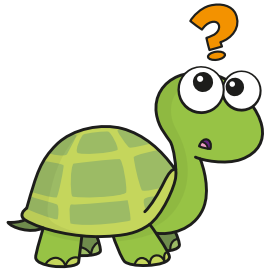
▶ $3\frac{5}{6}$ and $\frac{1}{12}$ ▶ $\frac{11}{18}$ and $2\frac{7}{9}$ ▶ $5\frac{5}{7}$ and $\frac{3}{14}$

Subtract from a mixed number

Reasoning and problem solving

Tiny is trying to work out this subtraction.

$$2\frac{5}{14} - \frac{2}{7}$$



$$2\frac{5}{14} - \frac{2}{7} = 2\frac{3}{7}$$

Do you agree with Tiny?

Explain your answer.

Work out the correct answer.

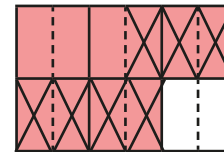


No

$$2\frac{1}{14}$$



Dora uses a diagram to work out a calculation.



What calculation is Dora working out?

What is the answer to the calculation?

$$1\frac{5}{6} - \frac{7}{12} \text{ or } 1\frac{10}{12} - \frac{7}{12}$$

$$1\frac{3}{12} \text{ or } 1\frac{1}{4}$$

Work out the subtraction.

$$5\frac{4}{9} - 3 - \frac{1}{3}$$

$$2\frac{1}{9}$$

Subtract from a mixed number – breaking the whole

Notes and guidance

There are many ways to subtract a fraction from a mixed number crossing the whole, and this small step encourages children to think flexibly about how to approach problems of this kind.

In addition to the methods illustrated in the Key learning section, children could also count back from the given fraction, providing the denominators are equal. This could be supported by the use of a number line. As in previous steps, either the denominators are equal, or one denominator is a multiple of the other.

Flexible partitioning and fluency in converting between improper fractions and mixed numbers are vital as children move from the pictorial to more abstract methods of recording their answers.

Things to look out for

- Children may not realise the need to cross the whole and instead subtract the smaller fraction part from the greater, for example working out $3\frac{1}{5} - \frac{4}{5}$ as $3\frac{4}{5} - \frac{1}{5}$ and giving the answer as $3\frac{3}{5}$
- Children may subtract numerators and denominators instead of finding a common denominator.
- Errors may occur with conversion between mixed numbers and improper fractions.

Key questions

- Which fraction is greater?
- How can you show the calculation as a diagram/on a number line?
- If the denominators are different, what do you need to do?
- How can you partition the mixed number? Is there more than one way?
- Is it easier to partition or to convert the mixed number to an improper fraction?
- Can you change the order of the numbers in a subtraction?

Possible sentence stems

- _____ can be partitioned into _____ and _____ or _____ and _____
- There are _____ in one whole, so there are _____ in _____

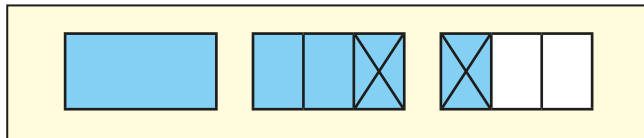
National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Subtract from a mixed number – breaking the whole

Key learning

- Kim uses diagrams to show that $2\frac{1}{3} - \frac{2}{3} = 1\frac{2}{3}$



Work out the subtractions.

- ▶ $4\frac{1}{4} - \frac{3}{4}$
- ▶ $3\frac{3}{8} - \frac{7}{8}$
- ▶ $2\frac{2}{5} - \frac{4}{5}$

- Scott and Esther use improper fractions to show that $2\frac{1}{3} - \frac{2}{3} = 1\frac{2}{3}$

Scott

$$\begin{aligned} 2\frac{1}{3} - \frac{2}{3} &= \frac{7}{3} - \frac{2}{3} \\ &= \frac{5}{3} \\ &= 1\frac{2}{3} \end{aligned}$$

Esther

$$\begin{aligned} 2\frac{1}{3} - \frac{2}{3} &= 1\frac{4}{3} - \frac{2}{3} \\ &= 1\frac{2}{3} \end{aligned}$$

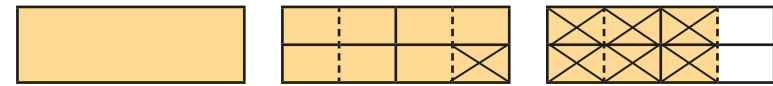
Explain why their methods work.

Whose method do you prefer?

Work out the subtractions.

- ▶ $3\frac{1}{5} - \frac{4}{5}$
- ▶ $2\frac{2}{9} - \frac{7}{9}$
- ▶ $3\frac{3}{7} - \frac{6}{7}$

- Here is a method for working out $2\frac{3}{4} - \frac{7}{8}$



Use this method to work out the subtractions.

- ▶ $3\frac{1}{3} - \frac{5}{6}$
- ▶ $4\frac{1}{5} - \frac{7}{10}$
- ▶ $5\frac{2}{3} - \frac{7}{9}$

- Tommy uses flexible partitioning to work out $6\frac{4}{9} - \frac{2}{3}$

$$6\frac{4}{9} - \frac{2}{3} = 5 + 1\frac{4}{9} - \frac{2}{3} = 5 + \frac{13}{9} - \frac{6}{9} = 5\frac{7}{9}$$

Use Tommy's method to work out the subtractions.

- ▶ $4\frac{2}{3} - \frac{5}{6}$
- ▶ $3\frac{1}{5} - \frac{7}{15}$
- ▶ $2\frac{1}{4} - \frac{7}{8}$

- Aisha has $3\frac{1}{4}$ bags of sugar.
She uses $\frac{7}{8}$ of a bag of sugar.
How much sugar does she have left?

Subtract from a mixed number – breaking the whole

Reasoning and problem solving

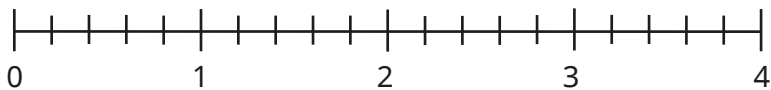
Write the digits 2, 3 and 4 in the boxes to make the calculation correct.

$$27\frac{1}{\square} - \frac{\square}{6} = 26\frac{\square}{3}$$

$$27\frac{1}{3} - \frac{4}{6} = 26\frac{2}{3}$$

Show how you can use a number line to

work out $2\frac{3}{5} - \frac{4}{5}$



any correct method that illustrates $2\frac{3}{5} - \frac{4}{5} = 1\frac{4}{5}$

Three children are working out $6\frac{2}{3} - \frac{5}{6}$

They all use partitioning to help them.



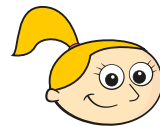
Annie

$$5 + 1\frac{2}{3} - \frac{5}{6}$$



Mo

$$5 + 1\frac{4}{6} - \frac{5}{6}$$



Eva

$$5 + \frac{10}{6} - \frac{5}{6}$$

Yes

Does each child have a correct starting point?

Explain your answer.

Subtract two mixed numbers

Notes and guidance

In this final small step of the block, children learn to subtract one mixed number from another.

Children begin by looking at simple cases where they partition two mixed numbers, then subtract the wholes and subtract the fractional parts. They then progress to more complex problems where they need to find a common denominator and/or break the whole.

As with earlier steps, there are a variety of possible approaches and these are explored, supported by diagrams. Children need to consider the most efficient approach for a given calculation rather than leaping into a method that might not be appropriate.

Things to look out for

- Children may assume they always need to convert to improper fractions, resulting in difficult arithmetic, when this is not necessary.
- Children may make errors when converting between mixed numbers and improper fractions.
- Children may subtract the denominators as well as the numerators.
- Children may subtract the fractions in the wrong order when they need to break the whole.

Key questions

- Is it possible to subtract the whole parts and fractional parts separately? Why or why not?
- Will you need to “break the whole”? Why or why not?
- Does making the whole numbers greater make the calculation more difficult? Why or why not?
- Is it easier to partition or to change the mixed number to an improper fraction?
- What diagrams could you use to support you?

Possible sentence stems

- The mixed numbers can be partitioned into a _____ part and a _____ part.
- When breaking the whole, the first number can be partitioned into _____ and _____

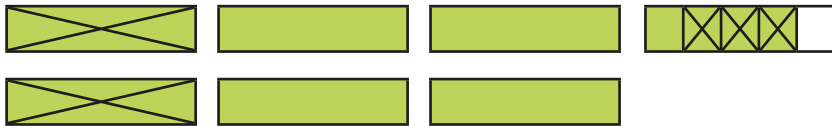
National Curriculum links

- Add and subtract fractions with the same denominator, and denominators that are multiples of the same number
- Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number

Subtract two mixed numbers

Key learning

- The diagram illustrates $6\frac{4}{5} - 2\frac{3}{5}$



$$6\frac{4}{5} - 2\frac{3}{5} = 4\frac{1}{5}$$

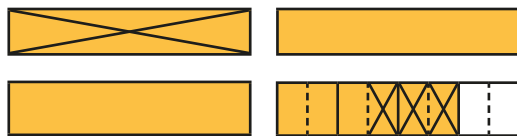
Work out the subtractions.

$$3\frac{5}{9} - 1\frac{1}{9}$$

$$6\frac{5}{7} - 4\frac{3}{7}$$

$$4\frac{13}{15} - 1\frac{2}{15}$$

- Here is a bar model to help work out $3\frac{3}{4} - 1\frac{3}{8}$



$$3\frac{3}{4} - 1\frac{3}{8} = 2\frac{3}{8}$$

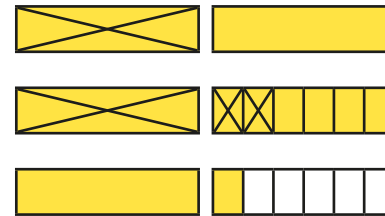
Work out the subtractions.

$$3\frac{7}{8} - 2\frac{3}{4}$$

$$5\frac{5}{6} - 2\frac{1}{3}$$

$$3\frac{2}{3} - 1\frac{5}{9}$$

- Here is a method for working out $5\frac{1}{6} - 2\frac{1}{3}$



$$5\frac{1}{6} - 2\frac{1}{3} = 4\frac{7}{6} - 2\frac{1}{3} = 4\frac{7}{6} - 2\frac{2}{6} = 2\frac{5}{6}$$



Use this method to work out the subtractions.

$$3\frac{1}{4} - 2\frac{5}{8}$$

$$5\frac{1}{3} - 2\frac{7}{12}$$

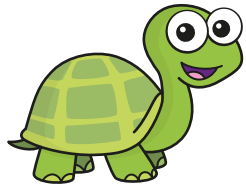
$$27\frac{1}{3} - 14\frac{7}{15}$$

- Compare the two methods for working out $5\frac{4}{15} - 1\frac{8}{15}$

Method 1	Method 2
$5\frac{4}{15} - 1\frac{8}{15} = 4\frac{19}{15} - 1\frac{8}{15}$ $= 3\frac{11}{15}$	$5\frac{4}{15} - 1\frac{8}{15} = \frac{79}{15} - \frac{23}{15}$ $= \frac{56}{15}$ $= 3\frac{11}{15}$

Subtract two mixed numbers

Reasoning and problem solving



$6 - 4\frac{3}{4} = 2\frac{3}{4}$

$1\frac{1}{4}$

Explain why Tiny is wrong.
Find the correct answer.



A bag of dog food contains only red, brown and orange biscuits.

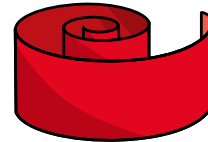


- The total mass of the dog food is 7 kg.
- The mass of red biscuits is $3\frac{3}{4}$ kg.
- The mass of brown biscuits is $1\frac{7}{16}$ kg.

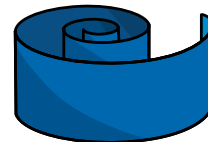
$1\frac{13}{16}$ kg

What is the mass of the orange biscuits?

Rosie has $20\frac{3}{4}$ cm of ribbon.



Annie has $6\frac{7}{8}$ cm less ribbon than Rosie has.



$13\frac{7}{8}$ cm

$34\frac{5}{8}$ cm

How much ribbon does Annie have?

How much ribbon do Annie and Rosie have altogether?

Compare methods with a partner.

